Pathway to A in CZ1001 Discrete Mathematics @Yuan3y

►**Group:**
Closure
Associativity (a#b)#c=a#(b#c)
Identity a#i=i#a=a
Invertibility a#b=i

►p → q ≡ ¬p ∨ q

►p **only if** q ≡ ¬q →¬p ≡ p→q

►When p → q (≡ ¬p ∨ q)
p is called a **sufficient** condition for q
q is a **necessary** condition for p

► "Everybody is cleverer than some monkey."
∀x∈H,∃y∈M,C(x,y)

► "Lions are fierce Animals"
∀x∈A (x is a lion → x is fierce)

► "Some fish climb trees"
∃x∈A (x is a fish ∧ x can climb trees)

►Mathematical Induction
[P(1) ∧∀k(P(k) →P(k+1))] → ∀n P(n)

►**Permutation**:
P(n,r)=n(n-1)(n-2)…(n-r+1)=n!/(n-r)!
Number of permutations of n objects taken r at a time

►**Combination**: C(n,r) = n!/r!/(n-r)!
Number of combinations of n objects taken r at a time, ordering not matter.

►The **conjugate** of z=a+bj is z'=a-bj

►**nth roots** of complex number:

Given z=r(cos θ+sin θj)=re^(j θ)

n√z=n√r * e^[j*(θ+2kPi)/n],
k∈{0..n-1}, 0<= θ<2Pi

►**Compute all nth roots of w:**
1. convert w into exponential form
   z^n=r*e^jθ
2.compute arg(z)
   arg(z^n)= θ+2kPi, k=0..n-1
   n arg(z)= θ+2kPi
   arg(z)= θ/n+2kPi/n
3.compute |z|
   |z^n|=r
   |z|=r^(1/n)
4.compute z
   z=|z|*e^(j*arg(z)), there are n-1 roots

►A m × n **matrix** = m rows, n columns,

```
a11  a12  …  a1j  …  a1n
a21  a22  …  a2j  …  a2n
…     …   …   …   …   …
ai1  ai2  …  aij  …  ain
…     …   …   …   …   …
am1  am2  …  amj  …  amn
```

aij is the element at i^{th} row and j^{th} column.

m×n **zero matrix**, 0mn

►**symmetric** matrix: A=AT

►**skew-symmetric** matrix: A=-AT

►A n×n matrix is a square matrix of **order** n.
The inverse, A^{-1} of a n×n square matrix A is an n×n square matrix s.t.
$AA^{-1}=A^{-1}A=I_n$

►**Row echelon form:**

The nonzero rows in A lie above all zero rows;
The first nonzero entry in a nonzero row (pivot) lies to the right of the pivot in the row immediately above it.

►**Reduced row echelon form:**
A is already in echelon form;
In every column containing a pivot, the pivot has value 1 and all other entries in the column are zero (IMOW: a row starts always with 1)

►**Gauss-Jordan Elimination:**
0.Principle: $A|I_n= I_n|A^{-1}$
1. Augment A with In to get $A|I_n$
2. Apply EROs to reduce left part to I_n
=> $I_n|A^{-1}$
3. The right part is A^{-1}

►**Solve linear system:**

0.Principle: A×X=B => $X=A^{-1}$×B
1.A|B
2.ERO to I|S
3.S is the solution

►**Identity relation** on A: I_A={(a,a)|a∈A}
If R⊆A×B, then the inverse of R,
R^{-1}={(b,a)|(a,b) ∈R}
R^{-1}⊆B×A

►Given R⊆A×B, S⊆B×C, the **composition** of R and S
R ∘ S={(a,c) ∈A×C|∃b∈B, aRb ∧ bSc}

►R is **reflexive**:
∀x∈A, xRx :I_A⊆R

►R is **symmetric**:
∀x, y∈A, (x,y)∈R→(y,x)∈R:R=R^{-1}

►R is **transitive**: ∀x, y,z∈A,
((x,y)∈R ∧ (y,z)∈R) →(x,z)∈R

►R is **antisymmetric**: ∀x, y∈A,
((x,y)∈R ∧ (y,x)∈R) →x=y

►**Equivalence** relation: **R**eflexive, **S**ymmetric, and **T**ransitive

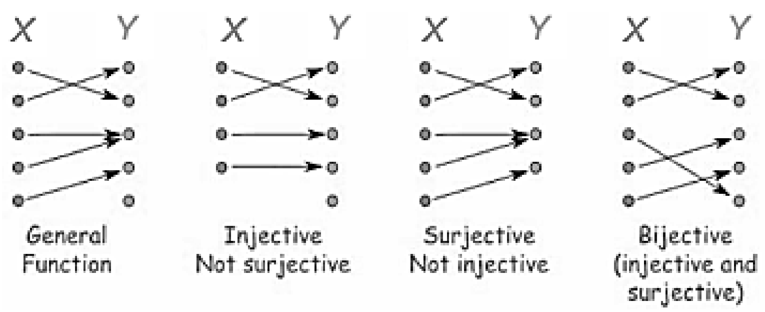
►**Partial order**: **R**eflexive, **A**ntisymmetric, and **T**ransitive

►**Function**: f:X→Y
iff for every x in X, there must be exactly one y in Y such that y = f(x).
(∀x∈X • ∃y∈Y • y=f(x))
∧(∀x1,x2∈X • x1=x2)
→ f(x1)=f(x2))

►**Injective, one-to-one**:
iff for every x in X, there is a y in Y such that f(x)=**y** and is **unique**.
∀x1,x2∈X • f(x1)=f(x2)
→x1=x2
|X|<=|Y|; Range⊆Y

►**Surjective, onto**:
iff for every y in Y, there is **at least one x** in X such that f(x)=y
∀y∈Y • ∃x∈X • f(x)=y
|X|>=|Y|; Range=Y

►**Bijective, one-to-one correspondence**
iff it is both **injective** and **surjective**.
|X|=|Y|;
Range=Y



►**Pigeonhole principle:**
given f:X→Y with |X|,|Y| both finite,
if |X|>|Y|, there is at least a y∈Y which is the image of at least 2 elements in X

►Applying pigeonhole principle:
1.find a function f:X→Y s.t.
$\exists x_i,x_j \in X \bullet x_i \neq x_j \wedge f(x_i)=f(x_j)$

►**Generalized pigeonhole** principle:
given f:X→Y with |X|,|Y| both finite,
if |X|>k*|Y|, there is at least one y∈Y which is the image of at least (k+1) distinct elements in X

►A **walk** is a finite alternating sequence of adjacent vertices and edges of G.

A **path** is a walk from v to w with no repeating edge.

A **simple path** is a path that with no repeated vertex other than the possibility that v=w.

A **closed walk** is a walk that starts and ends at the same vertex.

A **circuit** is a closed path.

A **simple circuit** or cycle is a closed simple path.

A **trivial circuit** is one with only one vertex and no edge.

A circuit is **non-trivial** if it has >=1 edge.

►A graph G is connected -> ∀v,w∈V(G), ∃a simple path connecting v,w .

If v,w∈V(G) -> if one edge is removed, then there still exists a path from v to w in G.

►A **Hamiltonian Circuit** of a graph G is a closed walk that contains:
1. Non-repeated edges forming a subset of E(G);
2. Non-repeated vertices, except the same start/end vertex, forming the FULL set of V(G).

►If a graph G contains a **Hamiltonian circuit**, then G must contain a connected subgraph H with the following properties:
1. V(H)=V(G),
2. |E(H)|=|V(H)|=|V(G)|,
3. ∀v∈V(H),deg(v)=2 in H.

►An **Euler path** from v to w is one that starts at v and ends at w, passes every vertex at least once (>=1), and traverse every edge of G only once (=1).

►An **Euler circuit** of graph G is a closed walk containing:
1. Non-repeating edges forming the full set E(G),
2. Possibly repeated vertices forming the full set V(G).

►DeMorgan's law:
~(p∧q) ≡ ~p ∨ ~q
~(p∨q) ≡ ~p ∧ ~q

Commutative:
p ∧ q ≡ q ∧ p
p ∨ q ≡ q ∨ p

Identity:
p ∧ T ≡ p
p ∨ F ≡ p

Universal bound:
p ∨ T ≡ T
p ∧ F ≡ F

Negation:
p ∧ ¯p ≡ F
p ∨ ¯p ≡ T

Double negation:
¬(¬p) ≡ p

Idempotent:
p ∧ p ≡ p
p ∨ p ≡ p

Absorption:
p ∨ (p ∧ q) ≡ p
p ∧ (p ∨ q) ≡ p

Associative:
(p∨q)∨r ≡ p∨(q∨r)
(p∧q)∧r ≡ p∧(q∧r)

Distributive:
p∧(q∨r) ≡ (p∧q)∨(p∧r)
p∨(q∧r) ≡ (p∨q)∧(p∨r)

Conversion Theorem:
p → q ≡ ¬p ∨ q

Modus ponens : method of affirming
p → q; p; ∴q

Modus tollens : method of denying
p → q; ¬q; ∴¬p

Conjunctive simplification : particularizing
p∧q; ∴p

Conjunctive addition : specializing
p; q; ∴p∧q

Disjunctive addition : generalization
p; ∴p∨q

Rule of contradiction
¬p→C; ∴p

Disjunctive Syllogism : case elimination
p∨q; ¬p; ∴q

Dilemma : case by case discussions
p∨q; p→r; q→r; ∴r

Hypothetical Syllogism: chain implication
p → q; q → r; ∴p→ r

►Universal Instantiation
∀x∈D,P(x)
∴P(c)

►Universal Generalization
P(c) for any arbitrary c from the domain
∴∀x∈D,P(x)

►Existential Instantiation
∃x∈D,P(x)
∴P(c) for some c

►Existential Generalization
P(c)
∴∃x∈D,P(x)

►Empty set: Ø, {}
A∩B'=A-B
(A-B)'=(A∩B')'=A'∪B

►Identity:
A∪Ø=A
A∩U=A

Domination:
A∪U=U
A∩Ø=Ø

Idempotent:
A∪A=A
A∩A=A

Double Complement:
A''=A

Commutative:
A∪B=B∪A
A∩B=B∩A

Associative:
A∪(B∪C)=(A∪B)∪C
A∩(B∩C)=(A∩B) ∩C

Distributive:
A∩(B∪C)=(A∩B)∪(A∩C)
A∪(B∩C)=(A∪B)∩(A∪C)

De Morgan's:
(A∪B)'=A'∩B'
(A∩B)'=A'∪B'

Identity:
A∪(A∩B)=A
A∩(A∪B)=A

Alternative Representation for set difference:
A-B=A∩B'

►Vertex v and w are e's **endpoints**

e **connects** v and w, i.e., e **incident** on both v and w.

v is **adjacent** to w and vice versa.

Two distinct edges are **adjacent** if both incident on a common vertex.

Two distinct edges with same end points are **parallel**.

When v=w -> e is a **loop**

A node without an incident edge from another node is **isolated**.

An **empty graph** has no vertex, no edge.

A **multi-graph** is one that has 2 or more edges joining some pair(s) of vertices.

A **simple graph** is one that has no loop nor parallel edges.

A **complete graph** with n vertices, Kn, is a simple graph that has every vertex connected to every other distinct vertex by an edge.

A **bipartite graph** is one whose vertices can be partitioned to 2 disjoint subsets V and W s.t. each edge only connects a v∈V and a w∈W.

► theorem: given functions $f : A \longrightarrow B$ and $g : B \longrightarrow C$. If both f and g are bijections, then $g \circ f$ is also a bijection.

proof: $g \circ f$ is onto: consider an arbitrarily chosen $c \in C$,

g is onto ⇒ $\exists b \in B$ s.t. $g(b) = c$
f is onto ⇒ $\exists a \in A$ s.t. $f(a) = b$
$\Rightarrow \forall c \in C \cdot \exists a \in A \cdot \exists b \in B \cdot f(a) = b \wedge g(b) = c$
$\Rightarrow \forall c \in C \cdot \exists a \in A \cdot g(f(a)) = c$

$\therefore \forall c \in C, \exists a \in A, g \circ f(a) = c$ $\quad \therefore g \circ f$ is onto

proof $g \circ f$ is 1-to-1: Pick any $a_1, a_2 \in A$ s.t. $a_1 \neq a_2$.

f is 1-to-1 ⇒ $f(a_1) \neq f(a_2)$ and g is 1-to-1 ⇒ $g(f(a_1)) \neq g(f(a_2))$

$\therefore \forall a_1, a_2 \in A$, if $a_1 \neq a_2$ then $g \circ f(a_1) \neq g \circ f(a_2)$

$\therefore g \circ f$ is 1-to-1.